

Surface Areas and Volumes

106. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. (Take $\pi = \frac{22}{7}$)

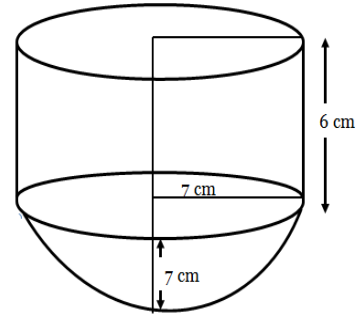
2011/2012/2015 [2 Marks]

Let h be the height and r be the base radius of the cylinder.

So, $h = 13 - 7 = 6$ cm

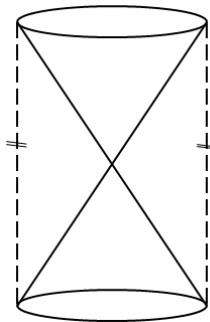
Therefore, inner surface area of the vessel

$$\begin{aligned}
 &= \text{Curved surface area of hemisphere} \\
 &\quad + \text{Curved surface area of cylinder} \\
 &= 2\pi r^2 + 2\pi rh \\
 &= 2\pi r[r + 2h] \\
 &= 2 \times \frac{22}{7} \times 7[7 + 6] \\
 &= 44 \times 13 \text{ cm}^2 = 572 \text{ cm}^2.
 \end{aligned}$$



107. An hour glass is made using identical double glass cones of diameter of 10 cm each. If total height is 24 cm, then find the surface area of the glass used in making it.

2014/2015 [3 Marks]



Radius of base of each cone = $\frac{10}{2}$ cm = 5 cm

and height of each cone = $\frac{24}{2}$ cm = 12 cm.

Let slant height of each cone be l .

So, from $l^2 = r^2 + h^2$, we have:

$$l^2 = 5^2 + 12^2$$

$$\Rightarrow l^2 = 169$$

$$\Rightarrow l = 13 \text{ cm}$$

So, curved surface area of one cone

$$= \pi r l = \pi \times 5 \times 13 = 65\pi \text{ cm}^2.$$

Surface area of base of one cone

$$= \pi r^2$$

$$= \pi \times (5)^2 = 25\pi \text{ cm}^2.$$

Therefore, surface area of the glass used

$$= (65\pi + 25\pi + 65\pi + 25\pi) \text{ cm}^2$$

$$= 180 \pi \text{ cm}^2.$$

108. Find the volume of an oil container, which is in the shape of a cylinder with hemispherical ends. Total length of the oil container is 23 metres and radius is 3 m.

2014/2015 [3 Marks]

Volume of the container = Volume of cylinder + Volume of hemispheres

$$= \pi r^2 h + \frac{2}{3} \pi r^3 + \frac{2}{3} \pi r^3$$

(Radii of cylinder and hemispheres are equal)

$$= \frac{22}{7} \times (3)^2 \times (23 - 3 - 3) + \frac{2}{3} \times \frac{22}{7} \times (3)^3 + \frac{2}{3} \times \frac{22}{7} \times (3)^3$$

$$= \left\{ \frac{22}{7} \times 9 \times 17 + \frac{4}{3} \times \frac{22}{7} \times 27 \right\} \text{ m}^3$$

$$= \frac{22}{7} \times 9(17 + 4) \text{ m}^3 = \frac{22}{7} \times 9 \times 21 \text{ m}^3$$

$$= 22 \times 27 \text{ m}^3 = 594 \text{ m}^3$$

109. The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 0.5 cm in diameter. A full barrel of ink in the pen can be used for writing 275 words on an average. How many words would be written using a bottle of ink containing one fourth of a litre?

2014/2015 [4Marks]

Volume of the fountain pen = $\pi r^2 h$

$$= \frac{22}{7} \times \left(\frac{0.5}{2}\right)^2 \times 7 \text{ cm}^3$$

$$= \frac{22}{7} \times 0.25 \times 0.25 \times 7 \text{ cm}^3$$

$$= \frac{25 \times 25 \times 25}{10000} \text{ cm}^3 = \frac{22}{16} \text{ cm}^3.$$

Now, in $\frac{22}{16} \text{ cm}^3$, number of words = 275

So, in $\frac{1}{4}$ litres, i.e., $\frac{1}{4} \times 1000 \text{ cm}^3$, in number of words

$$\begin{aligned} &= \frac{275 \times 16}{22} \times \frac{1000}{4} \\ &= 25 \times 2 \times 1000 \\ &= 5000 \text{ words} \end{aligned}$$

110. Three cubes of a metal whose edges are in the ratio 3 : 4 : 5 are melted and converted into a single cube whose diagonal is 12 cm. Find the edges of the three cubes.

2012/2015 [2 Marks]



Let the side of the new cube be 'a'.

So, $\sqrt{a^2 + a^2 + a^2} = 12\sqrt{3}$

$\Rightarrow a = 12$

Let the edges of the cubes be $3x$, $4x$, and $5x$.

Side of new cube = 12 cm

Volume of three cubes = Volume of single cube

$\Rightarrow (3x)^3 + (4x)^3 + (5x)^3 = 12 \times 12 \times 12$

$\Rightarrow 27x^3 + 64x^3 + 125x^3 = 12 \times 12 \times 12$

$\Rightarrow 216x^3 = 12 \times 12 \times 12$

$\Rightarrow x^3 = \frac{12 \times 12 \times 12}{216} = \left(\frac{12}{6}\right)^3$

$\Rightarrow x = \frac{12}{6} = 2.$

\therefore Edges of cubes are 6 cm, 8 cm, and 10 cm.
